MATHCOUNTS® Minis September 2016 Activity Solutions

Warm-Up!

1. We are told that $x = y + 3$ and $y = z - 5$, which can be rewritten as $y + 5 = z$. We are asked to determine the value of $z - x$. Substituting we get $(y + 5) - (y + 3) = y + 5 - y - 3 = 5 - 3 =$ **2**.

2. There are 3 cups of lemon juice and 7 cups of water or $3 + 7 = 10$ cups of liquid in the container. We want to add pure water so there is 25% lemon juice in the container. We know 25% is equivalent to 1/4. We can set up the equation $3/(10 + w) = 1/4$. Cross multiplying we get $12 = 10 + w$. So $w = 12 - 10 =$ **2** cups.

3. Juan takes a number: n, adds 2 to it: $n + 2$, multiplies the answer by 2: $(n + 2) \times 2 = 2n + 4$, subtracts 2 from the result: $2n + 4 - 2 = 2n + 2$ and divides that by 2: $(2n + 2) \div 2 = n + 1$. His answer is 7 so the original number must be $n = 7 - 1 =$ **6**.

4. We are told the television costs \$299 and the older sibling will pay \$45 more than the younger sibling. This means that the other $299 - 45 = 254$ dollars will be split evenly between the two siblings. Therefore, the younger sibling will pay $254 \div 2 =$ **127** dollars.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. Let the number of questions on the examination by q. If the student scores a 50%, that means $1/2 \times q$ questions were answered correctly. We know he answered 15 of the first 20 correct and 1/3 of the remaining or $1/3 \times (q - 20)$. So we have $15 + 1/3 \times (q - 20) = 1/2 \times q$. Multiplying both sides of the equation by 6, we get $90 + 2(q - 20) = 3q$. Solving for q, we get $90 + 2q - 40 = 3q$ and therefore $q =$ **50** questions.

6. We are told that Douglas' favorite number is a positive two-digit integer; let's call it n. Then a new number is created by adding a 7 digit to the end,this is the same as multiplying the number by 10 and adding 7 or $10n + 7$. We are told that the new number is 385 more than Douglas' favorite number. So we have $10n + 7 = n + 385$. Subtracting n and 7 from both sides yields $9n = 378$. Dividing both sides by 9 gives us $n =$ **42**.

7. Let's call Mary's age now m and her sister's age now s. Mary is seven years older than her sister so we can write the equation $m = s + 7$ or $s = m - 7$. In three years, Mary will be twice as old as her sister will be, so $m + 3 = 2(s + 3)$. Substituting $s = m - 7$ into the second equation, we get $m + 3 = 2(m - 7 + 3)$. Solving for m, we get $m + 3 = 2m - 8$ and $m =$ **11** years old.

8. Let M and J represent the initial amounts Mary and Joe have, respectively. The amount of money Mary has originally is \$2 more than Joe so $M = J + 2$. Joe pays \$1 less than 2/5 of the money he has for the T-shirt, and Mary pays 1/3 of the money she has. We know that they pay the same amount, so $2/5 \times J - 1 = 1/3 \times M$. If we multiply both sides of the second equation by 15, we get $6J - 15 = 5M$. If we multiply both sides of the first equation by 5, we get $5M = 5J + 10$. Subtracting the first equation from the second, we get $J - 25 = 0$ and $J =$ \$25. We know M is $25 + 2 =$ \$27. So, altogether, they originally had $25 + 27 =$ **\$52**.